3. STATIC EQUILIBRIUM

NEWTON'S 2ND LAW

scaler

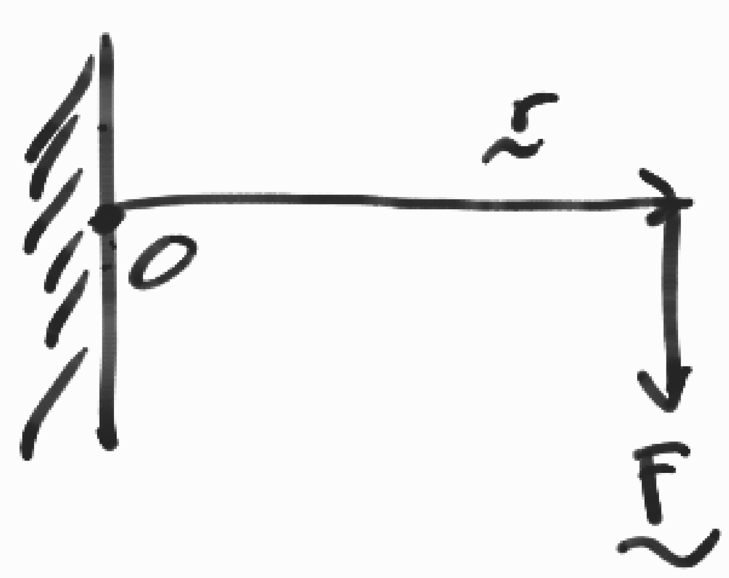
$$\underset{\sim}{F} = m\,\underset{\sim}{a} \qquad 3.1$$

vector vector

STATIC EQUILIBRIUM IS WHEN $\underset{\sim}{a} = \underset{\sim}{0}$

$$\Sigma \underset{\sim}{F} = \underset{\sim}{0} \qquad 3.2$$

<u>MOMENT</u> - TENDENCY OF A BODY TO ROTATE ABOUT A POINT DUE TO FORCES ACTING AT SOME DISTANCE FROM THE POINT



$$\underset{\sim}{M_o} = \underset{\sim}{r} \times \underset{\sim}{F} \qquad 3.3$$

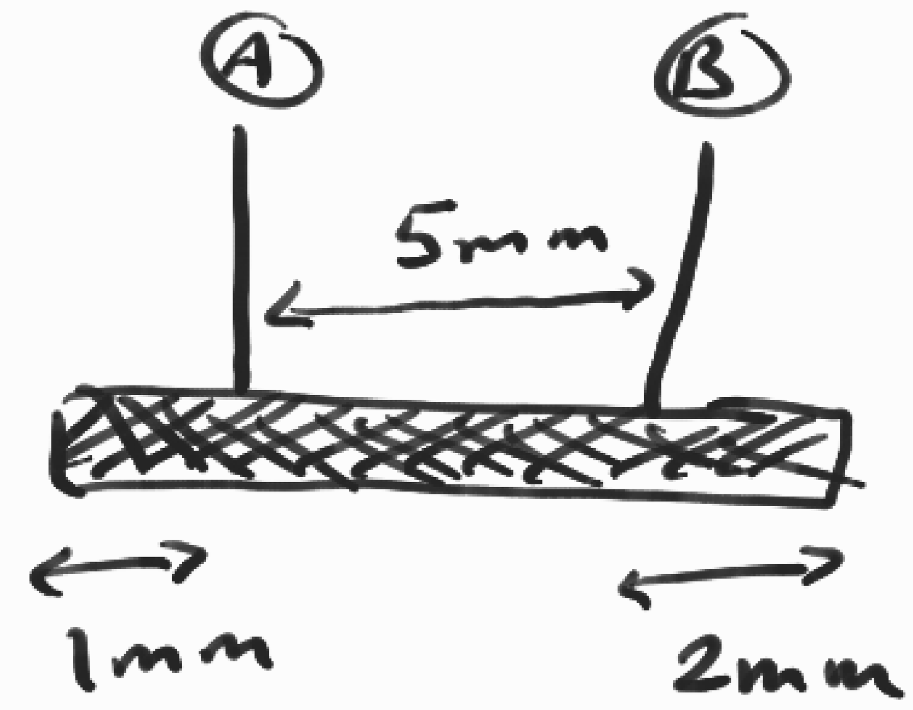
EQUILIBRIUM ALSO REQUIRES

$$\Sigma \underset{\sim}{M_o} = \underset{\sim}{0} \qquad 3.4$$

STEPS TO A STATICS PROBLEM

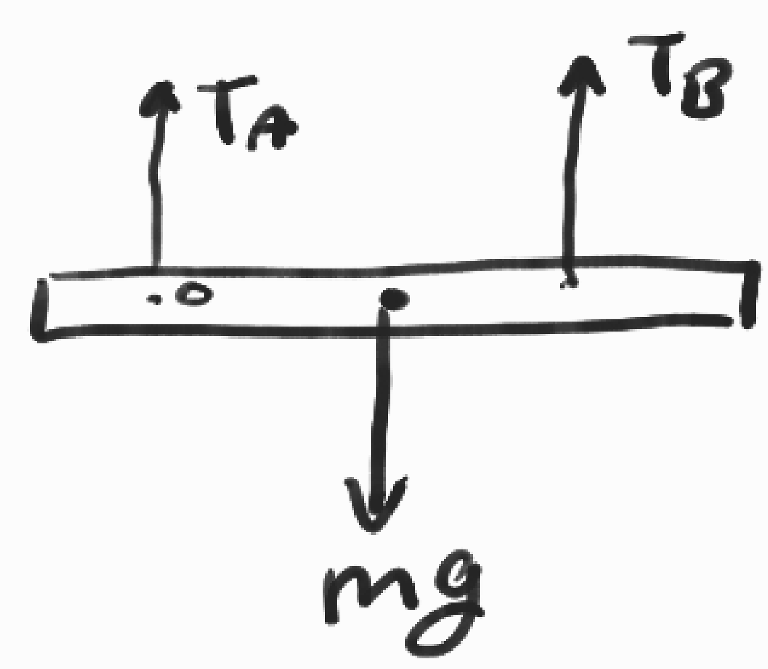
1) CHOOSE A COORDINATE SYSTEM
2) FREE BODY DIAGRAM
3) WRITE OUT THE EQL. EQUATIONS

EXAMPLE: CONSIDER THE BEAM BELOW, SUSPENDED BY 2 CABLES. IF BEAM HAS MASS m, WHAT IS TENSION IN EACH CABLE?



STEPS

1) 

2) FBD: 

3) EQL EQN ($\underset{\sim}{\Sigma F} = \underset{\sim}{\Sigma M} = \underset{\sim}{0}$)

$$\Sigma \underset{\sim}{F} = (T_A + T_B - mg)\,\underset{\sim}{e_2} = 0$$

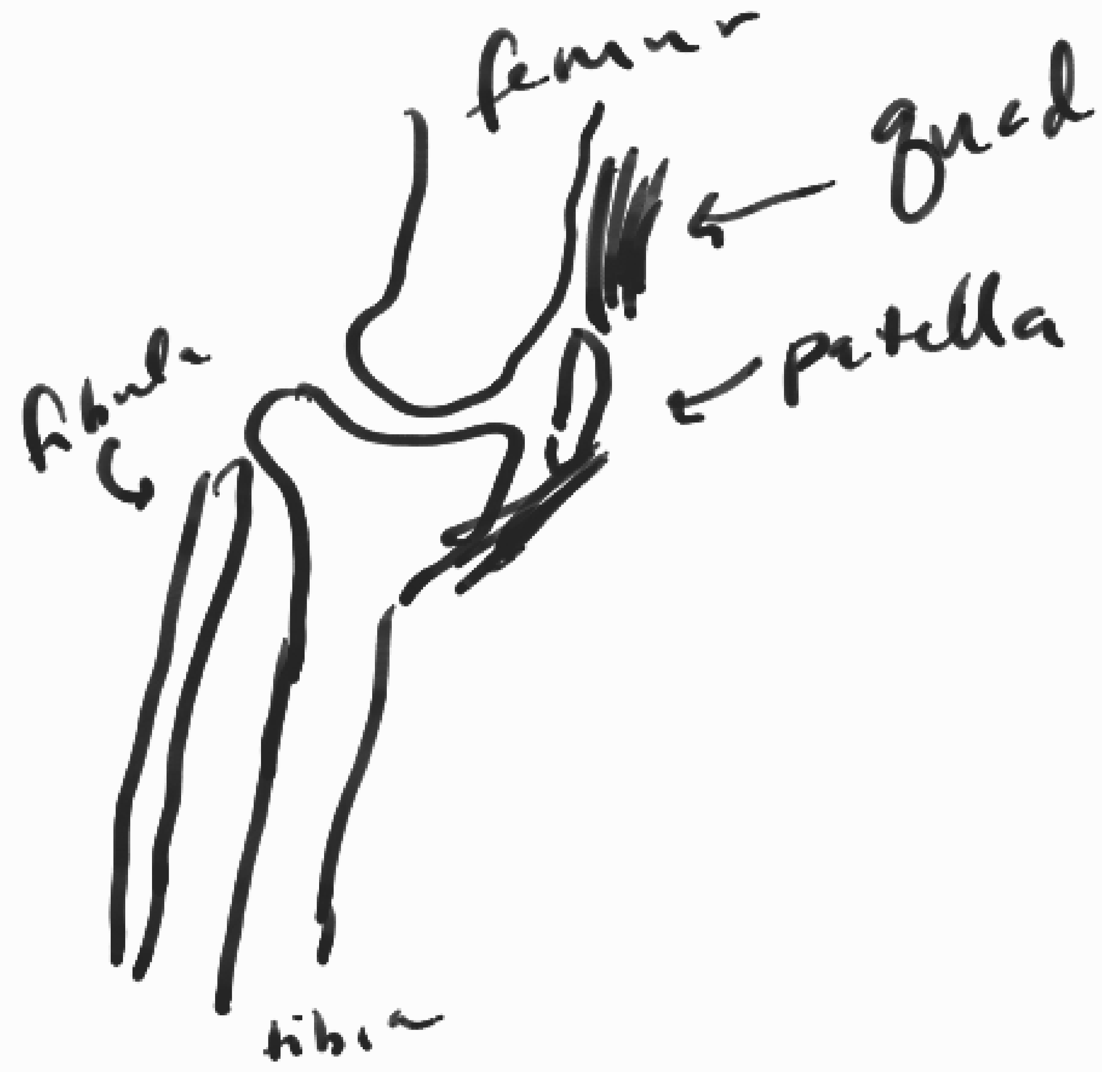
$$\Sigma \underset{\sim}{M_o} = (-3mg + 5T_B)\,\underset{\sim}{e_3} = 0$$

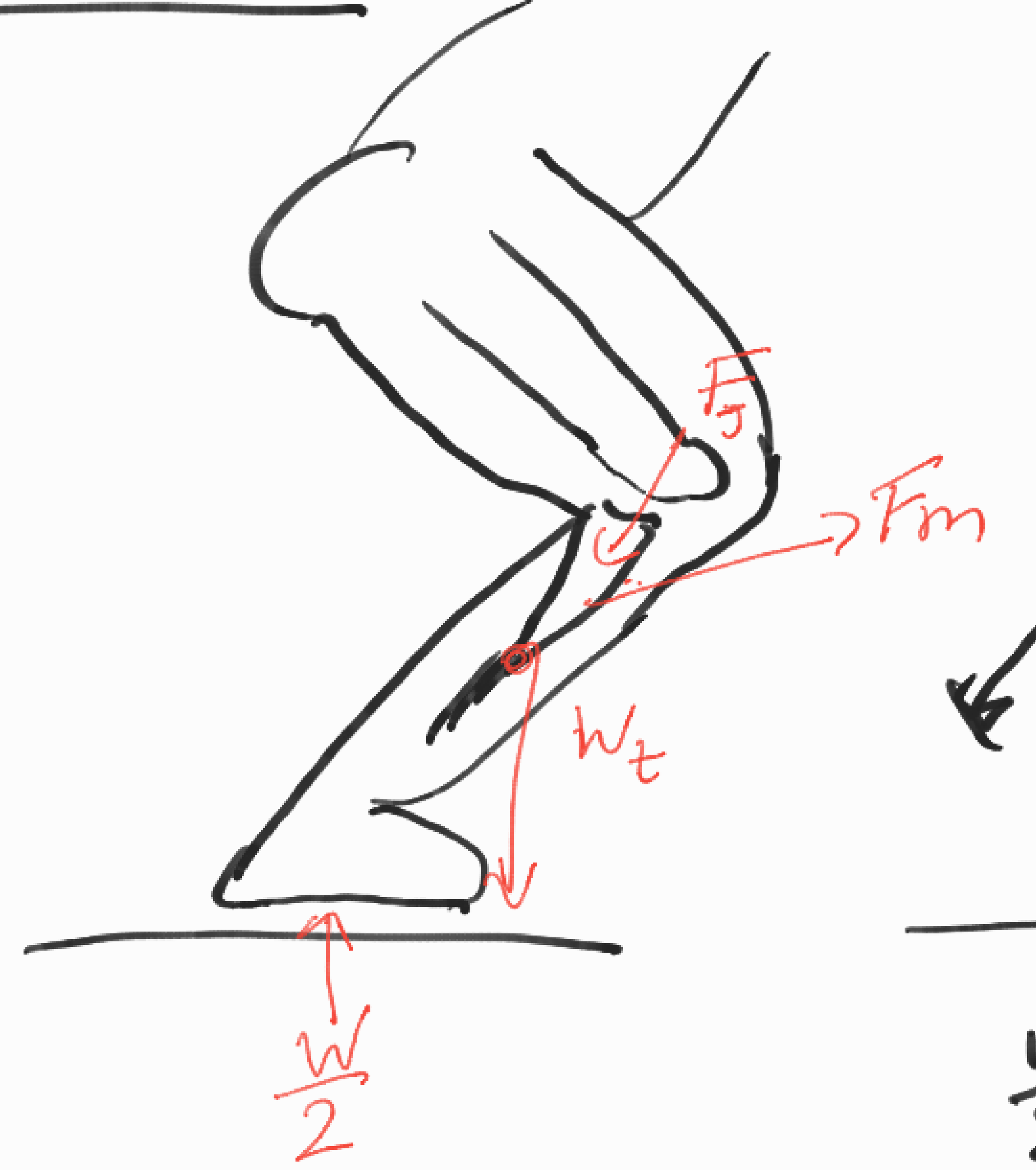
$$-5T_B = -3mg$$

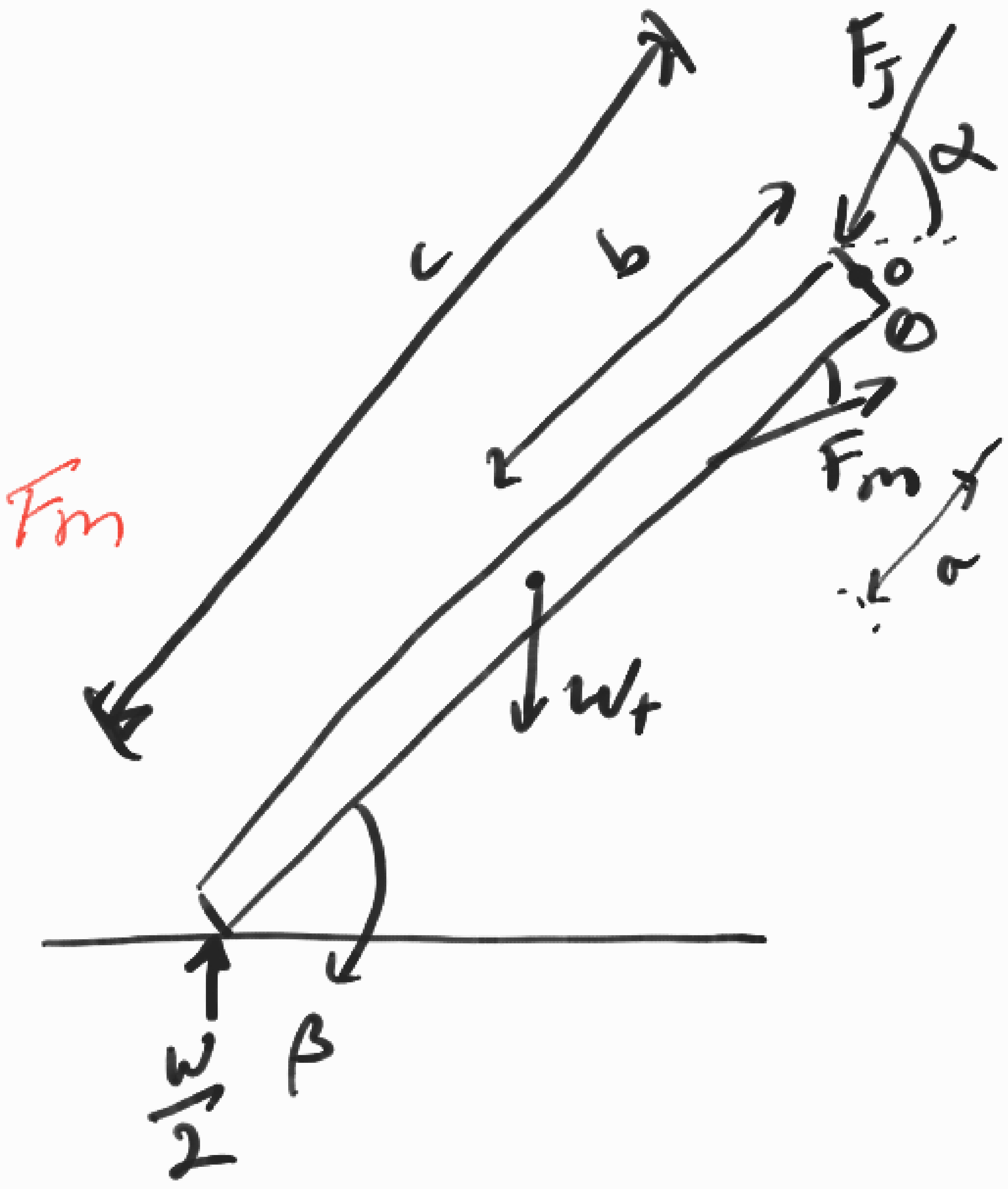
$$\boxed{T_B = \frac{3mg}{5}}$$

$$\boxed{T_A = mg - \frac{3mg}{5} = \frac{2mg}{5}}$$

EXAMPLE: KNEE JOINT







a = distance from knee to muscle insertion
b = distance from knee to centroid of tibia
c = length of tibia
α = joint reaction force angle
β = foot angle ($\beta = 90°$ full extension of knee
$\beta = 30°$ extreme flexion of ")
ϕ = muscle insertion angle
W_t = weight of tibia
W = body weight
F_m = muscle force
F_J = joint force

IF WE KNOW ANATOMY (a, b, c) + weight (W, W_t), AND WE CAN MEASURE β, WHAT ARE F_m, F_J, AND α?

1) L_{x_1}

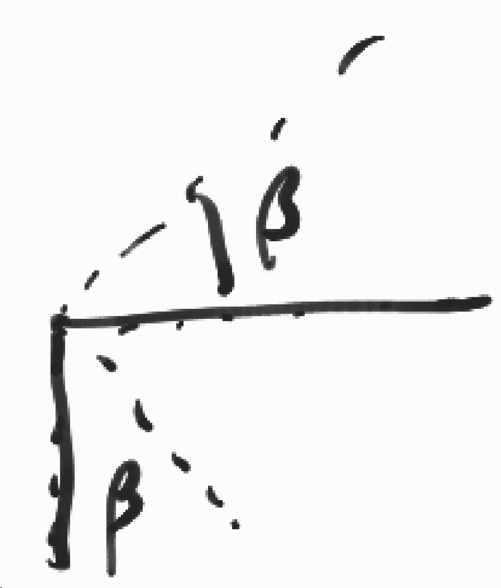
2) FBD - above

3) $\sum \underset{\sim}{F} = \underset{\sim}{0}$, $\sum \underset{\sim}{M_o} = \underset{\sim}{0}$ $\underset{\sim}{0}$ vs 0

$\hookrightarrow \sum \underset{\sim}{F} \cdot \underset{\sim}{e_1} = \sum \underset{\sim}{F} \cdot \underset{\sim}{e_2} = 0$ (3.5)

$$\sum \underset{\sim}{F} \cdot \underset{\sim}{e_1} = F_M \cos(\beta - \theta) - F_J \cos\alpha = 0 \quad (3.6)$$

$$\sum \underset{\sim}{F} \cdot \underset{\sim}{e_2} = \frac{W}{2} + F_m \sin(\beta - \theta) - w_t - F_J \sin\alpha \quad (3.7)$$



$$\sum \underset{\sim}{M_o} \cdot \underset{\sim}{e_3} = (F_M \sin\theta)a + (w_t \cos\beta)b - \left(\frac{W}{2}\cos\beta\right)c = 0 \quad 3.8$$

$\therefore$ 3.6 - 3.8 = 3 EQNS, AND F_m, F_J, α ARE 3 UNKNOWN
SO PLUG + CHUG!

SOLVE 3.8 FOR F_M FIRST B/C ONLY 1 UNKNOWN!

$$F_m = \frac{\left(\frac{Wc}{2} - w_t b\right)\cos\beta}{a \sin\theta} \quad 3.9$$

3.7 $\Rightarrow$ $$F_J \sin\alpha = \frac{W}{2} + F_m \sin(\beta - \theta) - w_t \quad 3.10$$

$$3.6 \Rightarrow \quad F_J \cos\alpha = F_m \cos(\beta - \theta) \qquad 3.11$$

$$3.10/3.11 \Rightarrow \quad \frac{\sin\alpha}{\cos\alpha} = \frac{\left(w/2 + F_m \sin(\beta-\theta) - W_t\right)}{F_m \cos(\beta - \theta)} \qquad 3.12$$

$$\alpha = \tan^{-1}\left[\qquad \right]$$

PLUG INTO 3.10 SOLVE FOR F_J. GOOGLE SAYS

$$\sin \tan^{-1}(\varphi) = \frac{\varphi}{\sqrt{1+\varphi^2}} \qquad 3.13$$

$$F_J = \sqrt{F_m^2 + \left(\frac{w}{2} - W_t\right)^2 + 2F_m\left(\frac{w}{2} - W_t\right)\sin(\beta - \theta)}$$

3.14